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# The influence of thermal expansion flow on droplet evaporation

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**Abstract**

It has been demonstrated recently that it follows from conservation of mass that unsteady temperature fields create flow in an incompressible fluid with a temperature-dependent density even in the absence of gravity. The paper studies the influence of thermal expansion flow on spherically symmetric evaporation of an isolated droplet. A model problem of a droplet evaporating at a constant rate is first considered. In this idealized situation one can use the assumption of a thin thermal boundary layer to solve analytically the unsteady moving-boundary heat conduction problem to find the temperature field inside the droplet both with and without the thermal expansion flow. Next evaporation of a fuel droplet in a diesel engine is studied numerically. The heat diffusion equation is solved in the liquid phase while the standard quasi-steady model is used for the gas phase. The results of the calculation show that for high ambient temperatures the influence of the thermal expansion flow on the droplet lifetime can be considerable.

*Keywords:* Droplet evaporation; Thermal expansion; Diesel fuel

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## 1. Introduction

It has been demonstrated recently by Yariv and Brenner (2004) that it follows from conservation of mass that unsteady temperature fields create flow in an incompressible fluid with a temperature-dependent density even in the absence of gravity. This thermal expansion flow has been overlooked so far because it is in most cases negligible. However, it can be important when temperature gradients are large and time scales are small. One example of such physical situation is evaporation of a droplet in hot environment.

The classical problem of droplet evaporation considers a liquid droplet in a spherically symmetric gas field (Lefebvre, 1989; Sirignano, 1999). Though it neglects the relative velocity between the gas and the droplet it is known to be a good approximation for small droplets, such as encountered in combustion chambers of liquid engines.

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The temperature profile inside an evaporating droplet depends on the ratio of liquid heating time to droplet life time (Sirignano, 1999). When this ratio is small, the liquid temperature can be considered uniform, as is often done. On the other hand, at very high ambient temperatures, as in combustion of liquid fuels, rapid evaporation may cause the droplet life time to be much shorter than the time for the droplet to heat through its interior. The droplet remains essentially cold with a large temperature gradient in a thin thermal boundary layer near its surface. It is in this situation that thermal expansion flow can be important.

Most droplet evaporation calculations use the value of liquid density corresponding to the droplet surface temperature (Lefebvre, 1989) but do not include further effects of thermal expansion. Sometimes the change in the radius of the droplet due to its thermal swelling is also taken into account (Bertoli and na Migliaccio, 1999).

The paper studies the influence of thermal expansion flow on spherically symmetric evaporation of an isolated droplet. A model problem of a droplet evaporating at a constant rate is first considered. In this idealized situation one can adopt the approach of Plesset and Zwick (1952) and use the assumption of a thin thermal boundary layer to solve analytically the unsteady moving-boundary heat conduction problem to find the temperature field inside the droplet both with and without the thermal expansion flow. The difference found between the two solutions suggests that the effect can be important also in realistic physical situations.

Next evaporation of a fuel droplet in a diesel engine is studied numerically. The heat diffusion equation is solved in the liquid phase while the standard quasi-steady model (Lefebvre, 1989; Sirignano, 1999) is used for the gas phase. The results of the calculation show that for high ambient temperatures the influence of the thermal expansion flow on the droplet lifetime can be considerable.

## 2. Basic equations

We begin by calculating the thermal expansion flow-field for an evaporating droplet. Following Yariv and Brenner (2004) we assume that the characteristic temperature difference of the problem $\Delta T$ and the thermal expansion coefficient $\beta$ satisfy[1]

$$\beta \Delta T \ll 1 \tag{1}$$

so the density–temperature relationship $\rho(T)$ can be linearized over a reference temperature $T_0$

$$\rho = \rho_0[1 - \beta(T - T_0)] \tag{2}$$

Here $\rho$ is the density, $T$ is the temperature and $\rho_0 = \rho(T_0)$.

The existence of the thermal expansion flow follows from the fact that the velocity field is coupled to the density field through the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{3}$$

Here $t$ is the time and $\mathbf{v}$ is the velocity vector.

Utilizing the fact that the flow velocity is small for small density variations, Yariv and Brenner (2004) linearized the continuity equation

$$\frac{\partial \rho}{\partial t} + \rho_0 \nabla \cdot \mathbf{v} = 0 \tag{4}$$

and used (2) to derive

$$\nabla \cdot \mathbf{v} = \beta \frac{\partial T}{\partial t} \tag{5}$$

It is shown by Yariv and Brenner (2004) that it follows from (5) that the net inward flux of heat creates the net outward flux of volume.

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[1] For example, for water at 300 K, $\beta \approx 2.8 \times 10^{-4}\ \mathrm{K}^{-1}$ and $\beta \Delta T < 0.15$ even for $\Delta T = 500$ K.

To calculate the thermal expansion flow from (5) one needs to find the temperature field from the energy equation. Yariv and Brenner (2004) demonstrated that in the linear approximation the convection, pressure and dissipation terms in the energy equation were negligible, the resulting form of the equation being

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \tag{6}$$

Here $\alpha$ is the thermal diffusivity at the reference state.

Consider now a spherically symmetric evaporating droplet. Re-writing (5) and (6) in the spherical coordinate system with the radial coordinate $r$ and eliminating $\frac{\partial T}{\partial t}$, one obtains

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(v - \beta\alpha\frac{\partial T}{\partial r}\right)\right] = 0 \tag{7}$$

which upon integration yields

$$v = \beta\alpha\frac{\partial T}{\partial r} \tag{8}$$

This is radial flow that is created in an evaporating droplet by thermal expansion. One takes it into account by adding the convection term to the energy equation which will then be

$$\frac{\partial T}{\partial t} = \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) - \beta\alpha\left(\frac{\partial T}{\partial r}\right)^2 \tag{9}$$

To estimate the order of the convection term, we write (9) in the dimensionless form

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{x^2}\frac{\partial}{\partial x}\left(x^2\frac{\partial \theta}{\partial x}\right) - \beta\Delta T\left(\frac{\partial \theta}{\partial x}\right)^2 \tag{10}$$

where $x = r/R_0$, $\tau = \alpha t/R_0^2$, $\theta = (T - T_0)/\Delta T$, and $R_0$ is the initial radius of the droplet.

One sees from (10) that the convection term is formally $O(\beta\Delta T)$ and it might seem that it is much less than the unsteady and conduction terms which are $O(1)$. However, it is proportional to the square of the temperature gradient. If the temperature gradient is large (for example, when there is a thin thermal boundary layer at the droplet surface) the thermal expansion flow term can be important.

The thermal expansion flow (8) is in the direction of the temperature gradient. It brings a relatively cold liquid from the interior of the droplet to its surface. This means that the thermal expansion flow cools the droplet surface, decreases evaporation rate and prolongs the droplet evaporation. We will now estimate the effect of the thermal expansion flow on droplet evaporation.

## 3. Model problem

We first consider a model problem of a droplet evaporating with the constant rate. In this idealized situation it is possible to calculate analytically the temperature field inside the droplet both with and without the thermal expansion flow.

Consider a liquid droplet with a uniform initial temperature $T_0$ that was suddenly immersed into its own vapor. The ambient pressure in the gas phase is assumed low, so the droplet will become superheated and begin evaporating. In this model problem we neglect the thermal conductivity of the vapor and assume that there is no external heat flux to the droplet. It uses the heat of the superheating for evaporation and cools down during the process.

Due to the low pressure of the gas phase the rate of evaporation will be high and one can expect a thin thermal boundary layer near the droplet surface if the thermal conductivity of the droplet liquid is sufficiently low. In addition, since the droplet was immersed suddenly, thermodynamic equilibrium at the droplet surface may not yet be reached and the evaporation rate must be calculated from the kinetic theory. It will be determined by the droplet surface temperature and the ambient pressure in the vapor (Ytrehus, 1997; Shusser et al., 2000).

As the droplet cools down during the evaporation its surface temperature decreases reducing the evaporation rate. On the other hand, any decrease in the ambient pressure during the evaporation will increase the evaporation rate. One can think of an idealized situation where the ambient pressure decreases during the evaporation in the exact way to keep the evaporation rate constant. We will assume in this model problem that the droplet evaporates with a constant rate $J$.

We will now calculate the change in the droplet surface temperature both without and with the thermal expansion flow. Since we assume a thin thermal boundary layer the effect of the volume expansion of the droplet is negligible in this problem.

Let the droplet liquid properties be density $\rho$, thermal conductivity $k$ and specific heat $c_p$. The droplet radius $R$ satisfies

$$\dot{R} \equiv \frac{\mathrm{d}R}{\mathrm{d}t} = -\frac{J}{\rho} \tag{11}$$

It should be noted that strictly speaking an additional term should be added to (11) because $\nabla\cdot\mathbf{v} \neq 0$ (see (5)). However, this term is negligible in the linear approximation, as will be demonstrated later.

The temperature field inside the droplet is found by solving

$$\frac{\partial T}{\partial t} = \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) \tag{12}$$

$$r = 0: \quad \frac{\partial T}{\partial r} = 0; \quad r = R: \quad \frac{\partial T}{\partial r} = -\frac{LJ}{k} \tag{13}$$

$$t = 0: \quad T = T_0 \tag{14}$$

Here $L$ is the heat of evaporation.

Since we are considering the situation when there is a large temperature gradient in a thin thermal boundary layer near the surface we will use the approach of Plesset and Zwick (1952) and assume that the temperature variations are appreciable only in the thermal boundary layer. Then taking a new variable

$$y = \frac{R^2}{r} - R \tag{15}$$

one obtains in the zero-order approximation ($r \approx R$)

$$\frac{\partial \Theta}{\partial t} - \frac{J}{\rho}\frac{\partial \Theta}{\partial y} = \alpha\frac{\partial^2 \Theta}{\partial y^2} \tag{16}$$

$$y \to \infty: \quad \frac{\partial \Theta}{\partial y} = 0; \quad y = 0: \quad \frac{\partial \Theta}{\partial y} = \frac{LJ}{k} \tag{17}$$

$$t = 0: \quad \Theta = 0 \tag{18}$$

Here $\Theta = T - T_0$.

Using Laplace transformation

$$\overline{\Theta} = \int_0^{\infty} \mathrm{e}^{-st}\Theta\,\mathrm{d}t \tag{19}$$

one obtains

$$\frac{\mathrm{d}^2\overline{\Theta}}{\mathrm{d}y^2} + \frac{J}{\rho\alpha}\frac{\partial \overline{\Theta}}{\partial y} - \frac{s\overline{\Theta}}{\alpha} = 0 \tag{20}$$

$$y \to \infty: \quad \frac{\mathrm{d}\overline{\Theta}}{\mathrm{d}y} = 0; \quad y = 0: \quad \frac{\mathrm{d}\overline{\Theta}}{\mathrm{d}y} = \frac{LJ}{ks} \tag{21}$$

with the solution

$$\overline{\Theta} = -\frac{LJ}{ks}\,\frac{\exp\left(-\frac{J}{2\rho\alpha}y\right)\exp\left(-y\sqrt{\frac{J^2}{4\rho^2\alpha^2}+\frac{s}{\alpha}}\right)}{\left(\frac{J}{2\rho\alpha}+\sqrt{\frac{J^2}{4\rho^2\alpha^2}+\frac{s}{\alpha}}\right)} \tag{22}$$

One can show that (22) corresponds to the following solution for the unsteady temperature field inside the droplet

$$T = T_0 + \frac{LJ}{k}\left[\frac{1}{2}\left(y+\frac{J}{\rho}t+\frac{\rho\alpha}{J}\right)\mathrm{erfc}\left(\frac{y+\frac{J}{\rho}t}{2\sqrt{\alpha t}}\right) - \frac{\rho\alpha}{2J}\mathrm{e}^{-\frac{J}{\rho\alpha}y}\mathrm{erfc}\left(\frac{y-\frac{J}{\rho}t}{2\sqrt{\alpha t}}\right) - \sqrt{\frac{\alpha t}{\pi}}\exp\left(-\frac{\left(y+\frac{J}{\rho}t\right)^2}{4\alpha t}\right)\right] \tag{23}$$

The surface temperature $T_\mathrm{s}$ is

$$T_\mathrm{s} = T_0 - \frac{L}{c_p} + \frac{LJ}{k}\left[\left(\frac{J}{2\rho}t+\frac{\rho\alpha}{J}\right)\mathrm{erfc}\left(\frac{J}{2\rho}\sqrt{\frac{t}{\alpha}}\right) - \sqrt{\frac{\alpha t}{\pi}}\mathrm{e}^{-\frac{J^2 t}{4\rho^2\alpha}}\right] \tag{24}$$

For high evaporation rates the value of the characteristic time for the surface temperature change $4\rho^2\alpha/J^2$ is rather small. Hence one can estimate the surface temperature drop caused by evaporation from the limit value of the surface temperature for $t\to\infty$, $T_\mathrm{sf}$

$$T_\mathrm{sf} = T_0 - \frac{L}{c_p} \tag{25}$$

We now estimate the droplet surface temperature perturbation caused by the thermal expansion flow (8). Writing the temperature as sum of its base value $T$ and its perturbation $T'$ and linearizing the energy equation (9) one obtains

$$\frac{\partial T'}{\partial t} = \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T'}{\partial r}\right) - \beta\alpha\left(\frac{\partial T}{\partial r}\right)^2 \tag{26}$$

$$r=0:\ \frac{\partial T'}{\partial r}=0;\quad r=R:\ \frac{\partial T'}{\partial r}=0 \tag{27}$$

$$t=0:\ T'=0 \tag{28}$$

Proceeding as previously, we use again the new variable (15) and assume a thin thermal boundary layer. In addition, since for high evaporation rates the temperature reaches its limit value very fast we approximate the temperature gradient in the base solution as its value for $t\to\infty$

$$\frac{\partial T}{\partial y} \approx \frac{LJ}{k}\mathrm{e}^{-\frac{J}{\rho\alpha}y} \tag{29}$$

We can then state the following problem for the temperature perturbation $T'$

$$\frac{\partial T'}{\partial t} + \dot{R}\frac{\partial T'}{\partial y} = \alpha\frac{\partial^2 T'}{\partial y^2} + Q\mathrm{e}^{-\frac{2J}{\rho\alpha}y} \tag{30}$$

$$y\to\infty:\ \frac{\partial T'}{\partial y}=0;\quad y=0:\ \frac{\partial T'}{\partial y}=0 \tag{31}$$

$$t=0:\ T'=0 \tag{32}$$

Here $Q = \frac{L^2J^2\beta\alpha}{k^2}$.

As was mentioned earlier, due to the presence of the thermal expansion flow an additional term must be added to (11) to calculate $\dot{R}$. Integrating the mass balance at the interface over the droplet surface one can show that this correction term is proportional to the integral of $\nabla \cdot \mathbf{v}$ over the droplet volume. This means that it is $O(\beta\Delta T)$ and since it is multiplied by $T'$ it can be neglected in the linear approximation.

We now substitute (11) into (30) and use Laplace transformation

$$\Theta' = \int_0^\infty \mathrm{e}^{-st} T' \, \mathrm{d}t \tag{33}$$

to obtain after solving the transformed problem

$$\Theta' = -\frac{Q\mathrm{e}^{-\frac{2J}{\rho\alpha}y}}{s\left(\frac{2J^2}{\rho^2\alpha} - s\right)} \left[1 - \frac{2J}{\rho\alpha} \frac{\exp\left(\frac{3J}{2\rho\alpha}y\right) \exp\left(-y\sqrt{\frac{J^2}{4\rho^2\alpha^2} + \frac{s}{\alpha}}\right)}{\left(\frac{J}{2\rho\alpha} + \sqrt{\frac{J^2}{4\rho^2\alpha^2} + \frac{s}{\alpha}}\right)}\right] \tag{34}$$

It is possible to invert the transformation to obtain the following solution for the temperature perturbation $T'$

$$\begin{aligned} T' = -\frac{Q\rho^2\alpha}{2J^2} \mathrm{e}^{-\frac{2J}{\rho\alpha}y} \Bigg[ & 1 - \mathrm{e}^{\frac{2J^2}{\rho^2\alpha}t} + \frac{2J}{\rho\alpha} \mathrm{e}^{\frac{J}{\rho\alpha}y} \times \Bigg\{ \frac{1}{2}\left(y + \frac{J}{\rho}t + \frac{\rho\alpha}{J}\right) \mathrm{erfc}\left(\frac{y + \frac{J}{\rho}t}{2\sqrt{\alpha t}}\right) - \frac{\rho\alpha}{2J} \mathrm{e}^{-\frac{J}{\rho\alpha}y} \mathrm{erfc}\left(\frac{y - \frac{J}{\rho}t}{2\sqrt{\alpha t}}\right) \\ & - \sqrt{\frac{\alpha t}{\pi}} \exp\left(-\frac{\left(y + \frac{J}{\rho}t\right)^2}{4\alpha t}\right) \Bigg\} + \mathrm{e}^{\frac{2J^2}{\rho^2\alpha}t} \left\{ \frac{1}{2} \mathrm{erfc}\left(\frac{y - \frac{3J}{\rho}t}{2\sqrt{\alpha t}}\right) - \mathrm{e}^{-\frac{3J}{\rho\alpha}y} \mathrm{erfc}\left(\frac{y + \frac{3J}{\rho}t}{2\sqrt{\alpha t}}\right) \right\} \\ & + \frac{1}{2} \mathrm{e}^{\frac{J}{2\rho\alpha}y} \mathrm{e}^{-\frac{J^2}{4\rho^2\alpha}t} \mathrm{erfc}\left(\frac{y + \frac{J}{\rho}t}{2\sqrt{\alpha t}}\right) \Bigg] \end{aligned} \tag{35}$$

Then the surface temperature perturbation $T'_{\mathrm{s}}$ will be

$$T'_{\mathrm{s}} = \frac{Q\rho^2\alpha}{2J^2} \left[1 - \left(2 + \frac{J^2}{\rho^2\alpha}t\right) \mathrm{erfc}\left(\frac{J}{2\rho}\sqrt{\frac{t}{\alpha}}\right) + \frac{2J}{\rho}\sqrt{\frac{t}{\pi\alpha}} \mathrm{e}^{-\frac{J^2}{4\rho^2\alpha}t} + \frac{3}{2} \mathrm{e}^{\frac{2J^2}{\rho^2\alpha}t} \mathrm{erfc}\left(\frac{3J}{2\rho}\sqrt{\frac{t}{\alpha}}\right) - \frac{1}{2} \mathrm{e}^{-\frac{J^2}{4\rho^2\alpha}t} \mathrm{erfc}\left(\frac{J}{2\rho}\sqrt{\frac{t}{\alpha}}\right)\right] \tag{36}$$

The limit value at $t \to \infty$, $T'_{\mathrm{sf}}$ is

$$T'_{\mathrm{sf}} = \frac{Q\rho^2\alpha}{2J^2} = \frac{\beta L^2}{2c_p^2} \tag{37}$$

One can estimate the influence of the thermal expansion flow in this model problem by comparing (25) and (37). The surface temperature drop caused by evaporation (25) must be corrected using (37) if the thermal expansion flow is taken into account. The relative magnitude of this correction is

$$\frac{T'_{\mathrm{sf}}}{T_0 - T_{\mathrm{sf}}} = \frac{\beta L}{2c_p} \tag{38}$$

Eq. (38) results in 8% correction for water at 300 K and about 10% correction for *n*-heptane fuel at the same temperature (see Lefebvre, 1989; Vargaftik, 1975 for the properties of *n*-heptane). This result suggests that thermal expansion flow can be important also in realistic physical situations. We now proceed to calculate the influence of thermal expansion flow on evaporation of a diesel fuel droplet.

## 4. Evaporation of a liquid fuel droplet

Consider a fuel droplet of initial radius $R_0$ and temperature $T_0$ immersed in a hot gas with the temperature $T_\infty$. To study the influence of the thermal expansion flow on the droplet evaporation we will modify the standard model of droplet evaporation to account for the thermal expansion flow effects.

### 4.1. Standard evaporation model

The standard model of droplet evaporation (Lefebvre, 1989; Sirignano, 1999) assumes that the fuel vapor is removed from the droplet surface by mass diffusion. In this model the droplet evaporation rate $J$ and the heat flux from the gas to the droplet $q$ are given by

$$J = \frac{k_\mathrm{g}}{c_{p\mathrm{g}}R}\ln(1+B) \tag{39}$$

$$q = \frac{k_\mathrm{g}}{R}\frac{\ln(1+B)}{B}(T_\infty - T_\mathrm{s}) \tag{40}$$

Here $c_{p\mathrm{g}}$, $k_\mathrm{g}$ are the specific heat and the heat conductivity of the gas and $B$ is the Spalding number

$$B = \frac{Y_\mathrm{fs}}{1-Y_\mathrm{fs}} \tag{41}$$

where $Y_\mathrm{fs}$ is the mass fraction of the fuel vapor at the droplet surface

$$Y_\mathrm{fs} = \frac{1}{1+\left(\frac{p}{p_\mathrm{s}}-1\right)\frac{M_\mathrm{a}}{M_\mathrm{f}}} \tag{42}$$

Here $p$ is the ambient pressure, $p_\mathrm{s}$ is the pressure of the saturated fuel vapor at the droplet surface $T_\mathrm{s}$, $M_\mathrm{a}$ and $M_\mathrm{f}$ are air and fuel molar masses, respectively. The saturated vapor pressure $p_\mathrm{s}$ can be estimated from the following modified form of the Clausius–Clapeyron equation (Lefebvre, 1989; Spiers, 1961)

$$p_\mathrm{s} = \exp\left(a - \frac{b}{T_\mathrm{s}-43}\right) \tag{43}$$

The values of $a$ and $b$ for specific fuels are given in Lefebvre (1989). The value of $p_\mathrm{s}$ given by (43) is in kPa.

The surface temperature $T_\mathrm{s}$ is found by solving the energy equation

$$\frac{\partial T}{\partial t} = \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) \tag{44}$$

The boundary condition at the droplet surface is obtained from the energy balance at the droplet surface

$$q = LJ + k\left.\frac{\partial T}{\partial r}\right|_{r=R} \tag{45}$$

Using (40) and (45) one obtains

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = \frac{k_\mathrm{g}}{kR}\ln(1+B)\left(\frac{T_\infty - T_\mathrm{s}}{B} - \frac{L}{c_{p\mathrm{g}}}\right) \tag{46}$$

The droplet radius satisfies

$$\frac{\mathrm{d}R}{\mathrm{d}t} = -\frac{J}{\rho_\mathrm{ls}} \tag{47}$$

where $\rho_\mathrm{ls}$ is the liquid density at the temperature $T_\mathrm{s}$

$$\rho_\mathrm{ls} = \rho[1-\beta(T_\mathrm{s}-T_0)] \tag{48}$$

### 4.2. Modified evaporation model

The following modifications must be made in the standard model to include the thermal expansion flow effects. First, the convective term must be added to the energy equation

$$\frac{\partial T}{\partial t}=\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)-\beta\alpha\left(\frac{\partial T}{\partial r}\right)^2 \tag{49}$$

In addition, the equation for the evolution of the droplet radius (47) should also be modified as it is written for zero liquid velocity at the droplet surface. To see this let us write the conservation of mass at the liquid side of the droplet–gas interface assuming that there is a non-zero liquid velocity at the droplet surface, say $U_1$

$$\rho_{\mathrm{ls}}\left(U_1-\frac{\mathrm{d}R}{\mathrm{d}t}\right)=J \tag{50}$$

It is seen from (50) that (47) is correct only for $U_1=0$.

To account for the thermal expansion flow we substitute (8) for $U_1$ to obtain instead of (47)

$$\frac{\mathrm{d}R}{\mathrm{d}t}=\beta\alpha\frac{\partial T}{\partial r}\bigg|_{r=R}-\frac{J}{\rho_{\mathrm{ls}}}=\beta\alpha\frac{k_{\mathrm{g}}}{kR}\ln(1+B)\left(\frac{T_\infty-T_{\mathrm{s}}}{B}-\frac{L}{c_{pg}}\right)-\frac{J}{\rho_{\mathrm{ls}}} \tag{51}$$

We now solve the above equations numerically for evaporation of a 50 μm droplet of the DF-2 fuel. The initial temperature of the droplet was $T_0=300$ K and the ambient conditions were $T_\infty=2000$ K, $p=3$ MPa. These conditions occur in practice when a fuel droplet is injected directly into the flame. Similar conditions were used by Sazhin et al. (2004) in their study of transient heating of diesel fuel droplets. The properties were (Lefebvre, 1989; Vargaftik, 1975; Sazhin et al., 2004) $M_{\mathrm{a}}=28.97$ kg/kmol, $M_{\mathrm{f}}=198$ kg/kmol, $\rho=846$ kg/m$^3$, $c_p=2000$ J/(kg K), $c_{pg}=2200$ J/(kg K), $k=0.14$ W/(m K), $k_{\mathrm{g}}=0.061$ W/(m K), $L=254$ kJ/kg, $\beta=8.3\times10^{-4}$ K$^{-1}$, $a=15.5274$, $b=5383.59$.

The finite-difference approximation of (44) and (49) was based on the Crank–Nicholson scheme which is implicit and of second-order in both space and time. To achieve fixed numerical domain in space, the equations were transformed to new dimensionless variables $\tau=\alpha t/R_0^2$, $z=r/R$. A uniform time step of $\Delta\tau=0.0001$ was used to advance the solution in time and a mesh of 201 nodes was used in space. Mesh and time-step refinement tests showed that the above values are satisfactory.

Figs. 1 and 2 show the evolution of the droplet radius and the droplet surface temperature with and without the thermal expansion flow. One can see from the plots that the influence of the thermal expansion flow on the

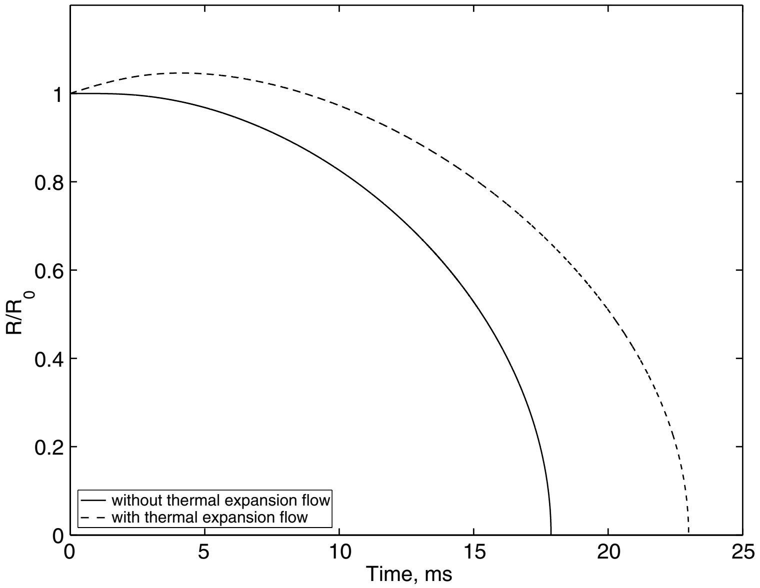

Fig. 1. The effect of the thermal expansion flow on the evolution of the droplet radius.

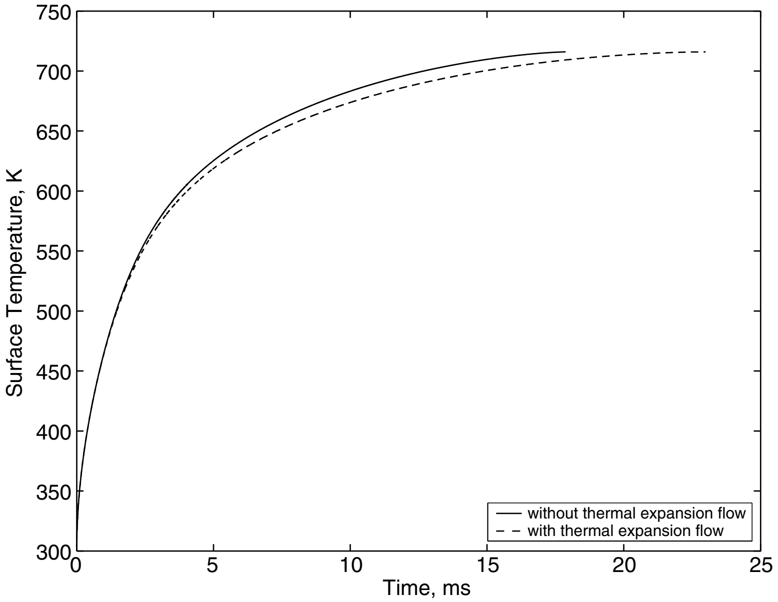

Fig. 2. The effect of the thermal expansion flow on the evolution of the droplet surface temperature.

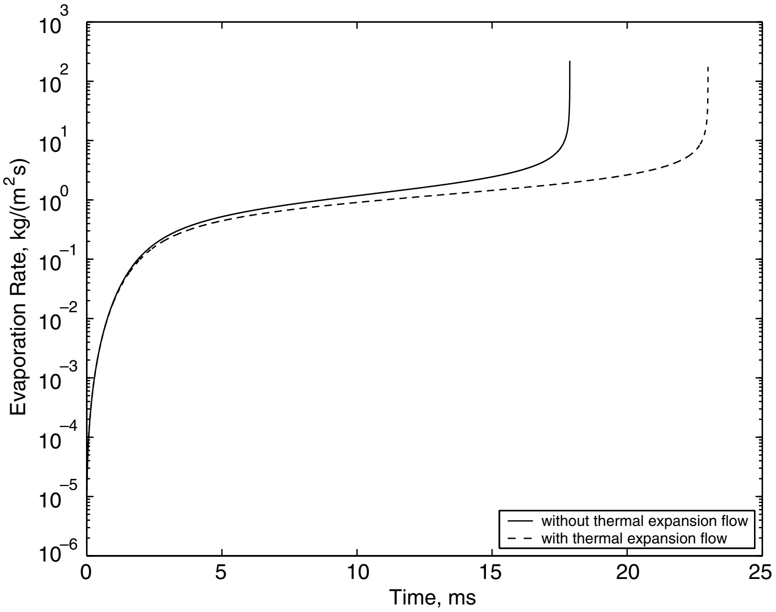

Fig. 3. The effect of the thermal expansion flow on the evolution of the evaporation rate.

droplet radius in this problem is considerable. Neglecting this phenomenon would cause an error of more than 20% in the droplet lifetime. On the other hand, the effect on the droplet surface temperature is smaller, the thermal expansion flow causing a slight decrease in its value. One can conclude from Figs. 1 and 2 that the influence of the correction in the equation for the droplet radius (51) is more important than the correction in the energy equation (49).

The influence of the thermal expansion flow on the evaporation rate can be seen from Fig. 3 where its evolution is plotted for both cases. One sees that at the beginning, when the evaporation rate is very low it

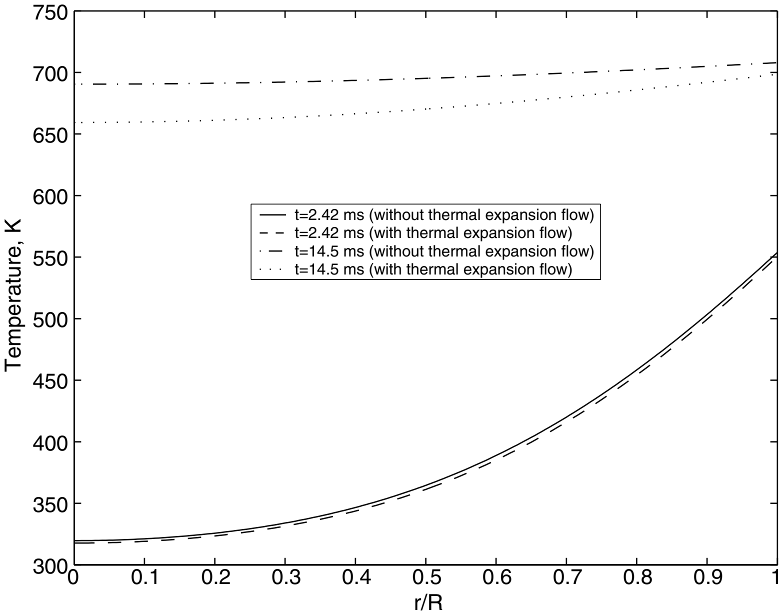

Fig. 4. Temperature profiles inside the droplet with and without the thermal expansion flow.

is practically the same both with and without the thermal expansion flow. Later, when the cooling effect of the thermal expansion flow accumulates, the difference between the evaporation rate values in both cases increases and becomes considerable after 15 ms. The sharp increase in the evaporation rate observed in both cases near the very end of the process is due to the droplet surface area approaching zero. It can be considered an artifact of the one-dimensional model and does not have any physical meaning.

We can see from Fig. 1 that at the beginning thermal expansion dominates and the droplet grows. Only after a while evaporation makes the droplet radius to decrease. The reason for this can be seen from Fig. 4 that shows the temperature profiles inside the droplet at two moments of time both with and without the thermal expansion flow. One sees from this figure that at early times there is a strong temperature gradient inside the droplet while at late times the droplet interior is already heated. Similar profiles were observed experimentally by Wong and Lin (1992). One can see from Fig. 4 how the thermal expansion flow delays heating of the droplet interior, as the difference between the profiles is largest at later stages in the interior of the droplet.

It is interesting to compare the results for droplet evaporation with the model problem considered in Section 3. For the model problem, we obtained the effect of the thermal expansion flow assuming the existence of a thin thermal boundary layer near the droplet surface. Though the existence of a thermal boundary layer is possible for evaporating droplets it does not exist in the physical situation considered here, as seen from Fig. 4. Nevertheless, we still obtained an effect of the thermal expansion flow. The temperature gradients at the early stages of evaporation were high enough to create strong thermal expansion flow due to which the thermal expansion dominated at the beginning of the evaporation.

## 5. Conclusions

We have studied the influence of the thermal expansion flow on evaporation of a liquid droplet. A model problem was considered and analytical solutions obtained suggested possible importance of this phenomenon. Evaporation of a fuel droplet at high ambient temperature was studied numerically. The results demonstrated that the thermal expansion flow can substantially prolong the process of droplet evaporation.

It should be noted that a one-dimensional, spherically symmetric model was used in this study. In most applications, the droplet will be moving relatively to the ambient gas and the viscous shear force at the interface will cause a circulating liquid flow within the droplet. However, the effect obtained in this work is sufficiently large to suggest that similar influence of the thermal expansion flow can be observed when more complicated models of evaporation are used. In addition, the effective-conductivity model of Abramzon and Sirignano (1989) gives an accurate description of the characteristic heating time and thermal inertia of the liquid in a moving evaporating droplet by using in the liquid phase the spherically symmetric heat diffusion equation with a modified thermal diffusivity. It is therefore suggested that the thermal expansion flow be taken into account when modeling droplet evaporation at high ambient temperatures.

This work used the description of the thermal expansion flow given by Yariv and Brenner (2004) who considered it as a linear perturbation. Since the thermal expansion flow was found to have considerable influence on droplet evaporation, it is plausible that non-linear effects could also be observed. One can suggest as a future work to calculate the thermal expansion flow by solving the full continuity equation coupled with the energy equation. This would allow a more precise estimation of the influence of the thermal expansion flow on droplet evaporation.

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